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To : See MAS Distribution List No. 2

Subject : STANAG 4106 LAND (EDITION 3) - PROCEDURES TO DETERMINE THE DEGREE OF BALLISTIC PERFORMANCE SIMILARITY OF NATO INDIRECT FIRE AMMUNITION AND THE APPLICABLE CORRECTIONS TO AIMING DATA

References :
a. MAS/60-LAND/4106 dated 7 March 1986 (Edition 2)
b. AC/225-D/1282 dated 11 October 1993 (Edition 3) (1st Draft)

Enclosure : STANAG 4106 (Edition 3)

1. The enclosed NATO Standardization Agreement which has been ratified by nations as reflected in page iii is promulgated herewith.

2. The references listed above are to be destroyed in accordance with local document destruction procedures.

3. AAP-4 should be amended to reflect the latest status of the STANAG.

ACTION BY NATIONAL STAFFS

4. National staffs are requested to examine page iii of the STANAG and if they have not already done so, to advise the Defence Support Division, IS, through their national delegation as appropriate of their intention regarding its ratification and implementation.

G.B. FERRARI
Major-General, ITAF
Chairman, MAS

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NATO UNCLASSIFIED

STANAG 4106
(Edition 3)

NORTH ATLANTIC TREATY ORGANIZATION

(N A T O)

MILITARY AGENCY FOR STANDARDIZATION

(M A S)

STANDARDIZATION AGREEMENT

SUBJECT : PROCEDURES TO DETERMINE THE DEGREE OF BALLISTIC PERFORMANCE SIMILARITY OF NATO INDIRECT FIRE AMMUNITION AND THE APPLICABLE CORRECTIONS TO AIMING DATA

Promulgated on 8 February 1995

G.B. FERRARI
Major-General, ITAF
Chairman, MAS

RECORD OF AMENDMENTS

No.	Reference/date of amendment	Date entered	Signature

EXPLANATORY NOTES

AGREEMENT

1. This NATO Standardization Agreement (STANAG) is promulgated by the Chairman MAS under the authority vested in him by the NATO Military Committee.

2. No departure may be made from the agreement without consultation with the tasking authority. Nations may propose changes at any time to the tasking authority where they will be processed in the same manner as the original agreement.

3. Ratifying nations have agreed that national orders, manuals and instructions implementing this STANAG will include a reference to the STANAG number for purposes of identification.

DEFINITIONS

4. Ratification is "The declaration by which a nation formally accepts the content of this Standardization Agreement".

5. Implementation is "The fulfilment by a nation of its obligations under this Standardization Agreement".

6. Reservation is "The stated qualification by a nation which describes that part of this Standardization Agreement which it cannot implement or can implement only with limitations".

RATIFICATION, IMPLEMENTATION AND RESERVATIONS

7. Page iii gives the details of ratification and implementation of this agreement. If no details are shown, it signifies that the nation has not yet notified the tasking authority of its intentions. Page iv (and subsequent) gives details of reservations and proprietary rights that have been stated.

ARMY

NATO STANDARDIZATION AGREEMENT
(STANAG)

PROCEDURES TO DETERMINE THE DEGREE OF BALLISTIC PERFORMANCE SIMILARITY OF NATO INDIRECT FIRE AMMUNITION AND THE APPLICABLE CORRECTIONS TO AIMING DATA

ANNEXES:
- A - Definitions
- B - Procedures for Fall of Shot Method
- C - Statistical Techniques for Fall of Shot Method
- D - Determination of the required sample size for Fall of Shot Method
- E - Procedure based on calibrated Modified Point Mass Trajectory and Radar Measurements
- F - Report on Firings

RELATED DOCUMENTS:

AC/225(Panel IV)D/147:	Technical Data for the Determination of Interchangeability of Artillery Weapons and Ammunition Systems.
STANAG 4119:	Adoption of a Standard Cannon Artillery Firing Table Format.
STANAG 4144:	Dynamic Firing Technique to Determine Ballistic Data for Cannon Artillery Firing Tables and Associated Fire Control Equipment.
STANAG 4355:	The Modified Point Mass Trajectory Model.
STANAG 4367:	Thermodynamic Interior Ballistic Model with Global Parameters.
STANAG 4400:	Derivation of Thermochemical Values for Interior Ballistic Calculations.
STANAG 4425:	A Procedure to Determine the Degree of Interchangeability of NATO Indirect Fire Ammunition.

AIM:

1. The aim of this agreement is to standardize procedures to determine the degree of similarity of the ballistic performance of NATO indirect fire weapon/ammunition systems and the applicable corrections to aiming data.

2. This agreement is not applicable to:

(a) direct fire weapon systems;

(b) surface to air weapon systems;

(c) pyrotechnic ammunition.

AGREEMENT:

3. Participating nations agree to use these procedures when comparing the ballistic performance of weapons and ammunition.

DEFINITIONS:

4. The definitions used in this STANAG are given in Annex A.

GENERAL:

5. This document specifies the NATO procedures to be used for comparing the ballistic performance of indirect fire weapon/ammunition systems. The intent of this document is to establish common procedures for determining whether the fire control system for one nation's weapon can be used effectively for fire control for another nation's weapon system and/or ammunition with or without corrections. These procedures include both a simulation approach when the user nation determines additional testing is unnecessary, and a test design and analysis approach using fall of shot and radar tracking data.

DETAILS OF THE AGREEMENT:

6. The details of the agreement are given hereunder and are divided into the following four parts:

(a) degrees of ballistic interchangeability among weapon/ammunition systems;

(b) assessment of ballistic differences between weapon/ammunition systems;

(c) responsibility for determining ballistic differences between weapon/ammunition systems;

(d) ballistic performance documentation of weapon/ammunition systems.

7. Degrees of Ballistic Interchangeability Among Weapon/Ammunition Systems. Four categories are established to assess the degree of ballistic interchangeability among weapon/ammunition systems:

(a) ballistic match;

(b) ballistic similitude within one percent of the range;

(c) ballistic similitude within five percent of the range;

(d) similitude not determined or greater than 5 percent of range.

As to operational aspects, refer to STANAG 4425.

8. Assessment of Ballistic Differences between Weapon/Ammunition Systems. Three procedures can be used to assess or quantify the degree of ballistic interchangeability. The first two procedures include limited firing

trials to estimate ballistic corrections and assign the degree of interchangeability. The third procedure relies on simulation techniques alone. The procedures are given below: for definitions, see Annex A.

(a) The Fall of Shot Procedure is described in Annex B. The preferred test plan (45 each of R and T configuration) is designed to compare both the mean ballistic performance and the round-to-round variability using standard statistical techniques. The minimum test plan (15 each of R and T configuration) is designed to address only the mean performance.

(b) The Radar Procedure is described in Annex E. These procedures take advantage of the availability of Doppler and/or position radars to help keep sample sizes small. The ability to provide more precise estimates of the actual aerodynamic forces has the added advantage of permitting better interpolation between charges. Only mean performance estimates are determined.

(c) The Simulation Procedure is also described in Annex E. Although no specific testing is prescribed, it is assumed that some form of firing with the T configuration has been conducted to demonstrate feasibility.

Mathematical details regarding statistical tests, sample sizes, etc. are found in Annexes C and D.

9. Responsibility for Determining Ballistic Differences between Weapon/Ammunition Systems. The responsibility for testing data may be assigned in two ways:

(a) If tests are required to obtain ballistic performance data for a particular weapon and ammunitioncombination, thenations involved agree to provide, subject to availability, the materiel or services required to complete the necessary testing and provide the data to AC/225(Panel IV/SP.2).

(b) A participating nation may agree to perform ballistic performance testing for a particular weapon and ammunition combination on behalf of AC/225(Panel IV/SP.2) and to provide the data to that sub-panel.

10. The costs and logistics of these exchanges are not a subject of this STANAG and are to be covered by agreements between participating nations.

11. Ballistic Difference Documentation of Weapon/Ammunition Systems. The format of the documentation for the ballistic corrections will be maintained and distributed to AC/225(Panel IV/SP.2) as described in Annex F. Although simple ballistic corrections are required for implementation into existing fire control systems, more complex corrections, for example corrections expressed as functions of elevation or Mach number, could be developed for fire control systems which are capable of accepting these charges in order to improve the quality of the aiming data.

IMPLEMENTATION OF THE AGREEMENT:

12. This STANAG is implemented when a nation uses the procedures to determine the ballistic differences between weapon/ammunition systems.

DEFINITIONS

Configuration:

A system consisting of the following components:

- weapon;
- propelling charge with primer, etc.;
- shell;
- fuze.

Reference (R) Configuration:

A configuration whose ballistic performance is well-known and for which a fire control system exists.

Test (T) Configuration:

The configuration whose ballistic performance is to be compared to the R-configuration. Components of the T-configuration may vary from those of the R-configuration by design or because of being manufactured by countries other than the country that designed or developed the components.

Fire Control System:

Any equipment used for determination of firing data for surface-to-surface artillery, usually in the form of firing tables or computers programmed with a certain mathematical model and a specific set of constants.

Ballistic Match:

(a) Interior Ballistic Match with respect to means:

The difference between the muzzle velocity of the interchanged ammunition and that of the national ammunition when they are fired from the national weapon, is not significantly greater, than to be expected between lots.

(b) External Ballistic Match with respect to means:

There is no significant difference between the impact of the interchanged ammunition and that of the national ammunition when they are fired from the national weapon, and corrections are made for differences in muzzle velocity and shell mass.

(c) Ballistic match with respect to variance:

The variance in muzzle velocity, range, deflection of the interchanged ammunition is not significantly greater than the variance of the national ammunition when they are fired from the national weapon.

Ballistic similitude:

In order to estimate ballistic corrections for both the range and deflection directions, the univariate, radial error in STANAG 4425 is divided into the two components shown below. This necessary refinement will not exceed the original radial errors specified in STANAG 4425.

The error between the theoretical (true) mean point of impact of the interchanged ammunition and that of the national ammunition when fired from the national weapon is within 1% of the range, with or without the application of a set of simple ballistic corrections to the aiming data for each charge to the nationally supplied ballistic data. The radial error will be within 1% of the range if the range and deflection components are within:

0.95% of the range in range, and
0.3% of the range in deflection.

Ballistic similitude 5%:

The error between the theoretical (true) mean point of impact of the interchanged ammunition and that of the national ammunition when fired from the national weapon is within 5% of the range, with or without the application of a set of corrections to the aiming data for each charge to the nationally supplied ballistic data. The radial error will be within 5% of the range if the range and deflection components are within:

4.75% of the range in range, and
1.5% of the range in deflection.

Simple ballistic corrections:

Corrections as indicated below:

A difference in	is corrected for by applying a correction to		Reference
	either	or	
Muzzle velocity	Muzzle velocity (meters/second)	Burning rate factor: f_β and down-tube resistance factor: f_R (percent)	STANAG 4367
Range	Air density (percent)	Form factor: i or Drag factor: f_D (percent)	STANAG 4355
Drift	Correction for drift (percent)	Lift factor: f_L (percent)	
Time of flight	Time of flight (seconds or percent)	Magnus force factor: Q_M (percent)	

Interchangeability:

A condition which exists when two or more items possess such functional and physical characteristics as to be equivalent in performance and durability, and are capable of being exchanged one for the other without alteration of the items themselves, or of adjoining items, except for adjustment, and without selection for fit and performance.

PROCEDURE FOR FALL OF SHOT METHOD

1. GENERAL

1.1 Standardization need not be thought of in terms of identity. For a common NATO equipment, standardization is achieved if the difference in ballistic performance of shell in different configurations is small enough to:

- permit the normal fire control procedures to be used without adjustment; or
- permit equivalent use through the introduction of simple ballistic corrections, common to all elevations for one and the same charge number.

1.2 The substance of the tests described in this document is to determine, by comparative firings, whether the ballistic performance of a shell in a test configuration (T) can be considered as being equal to the performance of a shell in a certain reference configuration (R) - if necessary by application of the above corrections.

The ballistic performances considered are:

- range;
- deflection;
- time of flight;
- muzzle velocity (MV).

1.3 The simple ballistic corrections considered are simple adjustments to the trajectory model (cf. STANAGs 4355 and 4367) for the reference configuration, common to all elevations for one and the same charge number.

2. JUSTIFICATION

The justification of the comparative firings is the assumption that when shells in two configurations are fired:

- on one and the same occasion;
- at the same elevation;
- in the same direction; and
- under nearly homogeneous conditions;

and the ranges and times of flight are adjusted to a common reference:

- shell mass; and
- muzzle velocity;

then:

- the difference in mean range, deflection, time of flight; and
- the ratio of variances;

are basically the same from occasion to occasion.

3. BALLISTIC MATCH AND SIMILITUDE

The T-configuration may be composed of T- or R-components as shown below:

Propelling charge	T	R	T	R
Shell	R	T	T	R
Fuze	R	R or T	R or T	T
See below, paragraph	3.1	3.2	3.3	3.4

3.1 Exchange of propelling charge only

Both interior and exterior ballistics may be relevant.

(a) Interior ballistic aspects

The MV to be treated statistically is the MV as measured and referred to R-shell standard mass.

Interior ballistic match with respect to variance indicates that the MV variance within lots of the T-configuration is not significantly greater than the variance within lots of the R-configuration. The variances of one T-lot and one R-lot are compared.

Interior ballistic match with respect to means indicates that the difference between the mean MV of a lot of the T-configuration and the mean MV of a lot of the R-configuration is not significantly greater than to be expected between lots of either configuration.

The test should be based on estimates of:

- variances within lots; and
- lot-to-lot variances.

The latter may be estimated by using charges from several lots, or may be provided from other sources.

(b) Exterior ballistic aspects

Exterior ballistic aspects should be added, if the different propelling charges have different burn characteristics, which could induce different yaw characteristics on the projectile, even when the muzzle velocities are the same.

Procedures described in paragraph 3.2(b) are to be followed.

3.2 Exchange of shell (and possibly fuze) only

Both interior and exterior ballistics are relevant.

(a) Interior ballistic aspects

The MV to be treated statistically is the MV as measured when firing one and the same propelling charge lot, and referred to R-shell standard mass. Differences in MV may occur due to differences in:

- chamber dimensions (shell dimensions);
- bore resistance.

Interior ballistic match with respect to variance indicates that the T-configuration MV variance is not significantly greater than the R-configuration MV variance.

Interior ballistic match with respect to means indicates that the T-configuration MV does not differ significantly from the R-configuration MV.

(b) Exterior ballistic aspects

Differences in drag, lift, Magnus force may occur.

Exterior ballistic match with respect to variance indicates that the T-configuration variances in range, deflection, time of flight are not significantly greater than the R-configuration variances, when referred to one and the same value of MV, and to the R-shell standard mass.

Exterior ballistic match with respect to means indicates that the T-configuration mean range, deflection, time of flight do not differ significantly from the R-configuration means, when referred to one and the same value of MV, and to the R-shell standard mass.

Similitude 1% indicates that the difference between the theoretical (true) mean points of impact of the T-shell and the R-shell is within:

- in range: 0.95% of the range;
- in deflection: 0.3% of the range;

with or without the application of a set of simple ballistic corrections to the aiming data for each charge to the nationally supplied ballistic data, and provided differences in MV are corrected for.

Similitude 5% indicates, that the difference between the theoretical (true) mean points of impact of the T-shell and the R-shell is within:

- in range: 4.75% of the range;
- in deflection: 1.5% of the range;

with or without the application of a set of simple ballistic corrections to the aiming data for each charge to the nationally supplied ballistic data, and provided differences in MV are corrected for.

3.3 Exchange of propelling charge and shell (and possibly fuze)

Both interior and exterior ballistics are relevant.

(a) Interior ballistic aspects

The MV to be treated statistically is the MV as measured and referred to R-shell standard mass. Differences in MV may occur, due to differences in:

- propelling charge characteristics;
- chamber dimensions (shell dimensions);
- bore resistance.

Interior ballistic match with respect to variance indicates that the MV variance within lots of the T-configuration is not significantly greater than the variance within lots of the R-configuration. The variances of one T-lot and one R-lot are compared.

Interior ballistic match with respect to means indicates that the difference between the mean MV of a lot of the T-configuration and the mean MV of a lot of the R-configuration is not significantly greater, than to be expected between lots of either configuration.

The test should be based on estimates of:

- variances within lots; and
- lot-to-lot variances.

The latter may be estimated by using charges from several lots, or may be provided from other sources.

(b) Exterior ballistic aspects

(As 3.2(b)).

3.4 Exchange of fuze only

Only exterior ballistics are relevant and only if the fuze deviates from the R-fuze with respect to mass or exterior dimensions (as 3.2(b)).

4. TEST DESIGN

4.1 Muzzle velocity levels

(a) For multi-charge weapons, firings should be carried out with specified charges, but in general three charges will be preferable:

- subsonic (subsonic velocity through the entire trajectory);
- transonic (muzzle velocity slightly above sound velocity, transition to subsonic velocity in the early part of the trajectory); and

- supersonic (muzzle velocity well above sound velocity, transition to subsonic velocity in the latter part of the trajectory, if ever).

The projectiles and propellant used in the test are to be conditioned to 21°C (or to the firing table standard, should some other standard be used in the firing table) for at least 24 hours immediately prior to firing (cf. STANAG 4144).

(b) If interior ballistic match has been assessed at two charges, the propelling charge of one zone being part of the propelling charge of the other, then interior ballistic match is assumed at possibly intervening charges also.

(c) If exterior ballistic match has been assessed at two charges of the same type, e.g. green bag (GB) or white bag (WB), single-perforated or multi-perforated, namely:

- a subsonic and a transonic charge; or
- a transonic and a supersonic charge; or
- two charges of the same velocity category (subsonic, transonic or supersonic);

then exterior ballistic match is assumed at any intervening charges of that type also.

(d) If ballistic similitude has been established at two charges of the same type, e.g. green bag (GB) or white bag (WB), single-perforated or multi-perforated, namely:

- a subsonic and a transonic charge; or
- a transonic and a supersonic charge; or
- two charges of the same velocity category (subsonic, transonic or supersonic)

applying the same corrections of both charges of that type, then similitude is assumed at any intervening charges of that type also, when the same corrections are applied.

4.2 Weapon

The tube should be in the first quarter of life (cf. STANAG 4144).

4.3 Proving Ground

The impact area should be level and approximately at the same height as the weapon.

4.4 Sample size

The number of usable rounds (excluding warmers and conditioners) to be fired with each combination of charge type/charge number/elevation to establish match or similitude will be determined by statistical considerations, cf. Annex C, and other considerations, such as economy, availability, time, etc.

If ballistic similitude (1% or 5%) with corrections is indicated, additional test firings may be required.

5. EXECUTION

5.1 For each charge, the two configurations (R and T) are fired:

- on one occasion; and
- in the same direction.

In general, the R-shells and the T-shells configurations should be fired alternately to eliminate the influence of varying ambient conditions during the firing.

However, it may be appropriate to fire in groups, if gun memory interference is suspected, e.g. in the case that two charge types are involved.

5.2 Ballistic Match

Interior ballistic match is tested at any elevation.

Exterior ballistic match is tested at an elevation of about 700 mils.

For each charge to be tested, the same number of rounds (usually 12, confine Annex C, figure 2) are fired with both configurations (R and T).

Ballistic match with respect to variance is assessed by comparing sample variances (R and T). The test is performed by applying the Levene-test or the F-test (one-sided significance tests).

Ballistic match with respect to means is assessed, using the t-test for the performance differences within R-T-pairs (pairs consisting of one R-round and one T-round) (two-sided significance test).

5.3 Ballistic Similitude

For each charge to be considered, ballistic similitude is tested by regression analysis of performance differences within R-T-pairs (pairs consisting of one R-round and one T-round).

In the usual procedure, five pairs are fired at each of three elevations, normally 400 mils, 700 mils, and 1100 mils.

In the smallest acceptable design, one pair is fired at each of five elevations, normally 350 mils, 550 mils, 750 mils, 950 mils and 1150 mils.

5.4 The required data for each test and reference shell are:

(1) meterological data;
(2) elevation;
(3) projectile mass;
(4) muzzle velocity;
(5) range;
(6) height of impact or burst relative to the weapon;
(7) deflection;
(8) time of flight.

As to the required accuracies, see STANAG 4144.

5.5 The quantities to be treated statistically, are test and reference configuration:

- muzzle velocity:
  as observed, and adjusted to a common projectile mass (the R-standard),
- range and
- time of flight:
  as observed, and adjusted to a common
  - projectile mass (the R-standard),
  - muzzle velocity (the R-standard), and
  - height of burst (impact), preferably the height of the weapon, and
  - deflection:
    as observed

For both configurations, the above adjustments are made by means of the reference firing table. (For use of firing tables, see Note).

6. OUTLIERS

6.1 It may happen, that an observation deviates that much from other observations, that it is felt to give an incorrect impression of the actual statistical distribution, either:

- because, although belonging to the population under investigation, an observation that deviates that much or more is

  so seldom occurring, that it would have an undue influence on the calculation of the estimate of the variance or the mean; or

- because the deviation might be due to some undetected error in the execution of the test.

Several rules exist for classifying outliers. One such rule is indicated in Annex C.

6.2 However, "outlying" observations may represent a characteristic of the particular configuration. Such observations, consequently, should be rejected with great discretion.

Note

Firing tables application for reduction to standard muzzle velocity and standard projectile mass.

1. The effect of deviations in:

- muzzle velocity (V_o); and
- projectile mass (m)

on range can be expressed by:

$\Delta X = (\Delta X / \Delta v_o) \; (\Delta v_o)_m \; + \; (\Delta X / \Delta m) \; . \; \Delta m,$

where

$(\Delta v_o)_m$ is the deviation from standard v_o, which would have been achieved, if the projectile mass were standard, indicated in m/s,

Δm is the deviation from projectile standard mass, indicated in %,

$(\Delta X/\Delta v_o)$ is the differential effect on range, caused by 1 m/s increase in v_o, which can be extracted directly from table F in the tabular FT,

$(\Delta X/\Delta m)$ is the differential effect on range, caused by 1% increase in projectile mass, determined by

$$(\Delta X/\Delta m) = (\Delta X/\Delta m)_{1SQ} \cdot \frac{\text{standard mass}}{(1\ \text{SQ mass}) \cdot 100},$$

where

$(\Delta X/\Delta m)_{1SQ}$ is the differential effect on range, caused by 1 SQ increase in projectile mass, which can be extracted directly from table F in the tabular FT.

2. Because the effect of an increase in projectile mass is both:

- a decrease in v_o; and
- a decrease in retardation due to air resistance, then the latter differential effect can be expressed by:

$$(\Delta X/\Delta m) = (\Delta X/\Delta v_o)\ (\Delta v_o/\Delta m) + (\Delta X/\Delta m)_{v_o};$$

where:

$(\Delta v_o/\Delta m)$ is the differential effect on v_o, caused by 1% increase in projectile mass;

$(\Delta x/\Delta m)_{v_o}$ is the differential effect on range, caused by 1% increase in projectile mass, provided v_o is constant.

3. Because of:

$$(\Delta X/\Delta m)_{v_o} = -(\Delta X/\Delta \rho);$$

which is the differential effect on X, caused by 1% decrease in air density, this differential effect can be extracted directly from the corresponding correction columns in the tabular FT.

Then $(\Delta v_o/\Delta m)$ can be calculated as

$$(\Delta v_o/\Delta m) = \frac{(\Delta X/\Delta m) - (\Delta X/\Delta m)_{v_o}}{(\Delta X/\Delta v_o)} = \frac{(\Delta X/\Delta m) + (\Delta X/\Delta \rho)}{(\Delta X/\Delta v_o)}$$

4. Consequently, the effect on range can be expressed by

$$\Delta X = (\Delta X/\Delta v_o) \left[(\Delta v_o)_m + (\Delta v_o/\Delta m) \cdot \Delta m \right] - (\Delta X/\Delta \rho) \cdot \Delta m,$$

where the effects of a deviation in projectile mass have been separated.

The total deviation in v_o is

$$\Delta v_o = (\Delta v_o)_m + (\Delta v_o/\Delta m) \cdot \Delta m.$$

This is the deviation that is measured.

STATISTICAL TECHNIQUES FOR FALL OF SHOT METHOD

1. INTRODUCTION

The aim of this Annex is to describe the statistical techniques to be used for assessment of ballistic match or ballistic similitude, associated with the fall of shot method.

2. BALLISTIC MATCH

2.1 Introduction

The number of rounds considered is

n_T of the T-configuration (index "T") and
n_R of the R-configuration (index "R").

Normally $n_T = n_R$ (= n).

The statistic variable X is muzzle velocity, range, deflection, time of flight, adjusted as described in Annex B.

The individual event is X_{Ti} or X_{Ri}
(i = 1, 2, ..., n_T or n_R).

2.2 Ballistic match with respect to variance

The true variances or population variances are

σ_T^2 and σ_R^2

The sample variances or the estimates of the population variances are

$$s_T^2 = \frac{1}{n_T - 1} \sum (x_{Ti} - \bar{x}_T)^2 \quad \text{and}$$

$$s_R^2 = \frac{1}{n_R - 1} \sum (x_{Ri} - \bar{x}_R)^2$$

where

$$\bar{x}_T = \frac{1}{n_T} \sum x_{Ti} \quad \text{and}$$

$$\bar{x}_R = \frac{1}{n_R} \sum x_{Ri}$$

are the simple means or the estimates of the population means μ_T and μ_R.

The hypothesis to be tested is H_0: $\sigma_T/\sigma_R \leq 1$ against H_1: $\sigma_T/\sigma_R > 1$, the significance level α = 0,05.

The variance is tested by either the Levene-test or the F-test. The Leven-test (*) is preferred, because it is less sensitive to departure from the normal distribution.

(*) Levene, H.: Robust tests for quality of variances, in: contributions to probability and statistics, essay in honour of H. Hotelling, Ed. by I. Olkin et al., Stanford 1960, pages 278-292.

2.2.1 The Levene-test (one-sided)

For both samples the medians (med_T and med_R) are determined. The absolute deviations of the i'th observation from the respective sample medians are:

$$d_{Ti} = | x_{Ti} - med_T | \text{ and}$$

$$d_{Ri} = | x_{Ri} - med_R | .$$

The sample means of these absolute deviations are

$$\bar{d}_T = \frac{1}{n_T} \cdot \sum d_{Ti} \text{ and}$$

$$\bar{d}_R = \frac{1}{n_R} \cdot \sum d_{Ri} .$$

It is assumed, that for practical purposes, if $\sigma_T = \sigma_R$, then the quotient

$$t' = \frac{\bar{d}_T - \bar{d}_R}{\sqrt{M \left[\frac{1}{n_T} + \frac{1}{n_R}\right]}}$$

is statistically distributed as t(f), the number of degrees of freedom being:

$$f = n_T + n_R - 2,$$

and

$$M = \frac{1}{n_T + n_R - 2} \cdot \left[\sum (d_{Ti} - \bar{d}_T)^2 + \sum (d_{Ri} - \bar{d}_R)^2\right]$$

is a pooled estimate of the moment of second order about the mean absolute deviations from the medians.

The hypothesis H_0 is accepted, if $t' < t_\alpha$, and otherwise rejected.

To achieve the same power of the test, the number of observations should be about 15% higher than the number, that would have been required for the F-test.

2.2.2 The F-test (one-sided)

(a) The quotient $(s_T/s_R)^2$ is computed. In case of normal distribution, if $\sigma_T = \sigma_R$, then this quotient follows the F-distribution, the numbers of degrees of freedom being

$$(f_1, f_2) = ((n_t - 1), (n_R - 1)).$$

The hypothesis Ho is accepted, if $F \leq F\alpha$, and otherwise rejected.

(b) The probability β of accepting Ho as a function of $\lambda = \sigma_T/\sigma_R$ and the number of observations ($n_T = n_R = n$) can be read from the OC curves in figure 1. For a specific value of λ, the power of the test is $1 - \beta$.

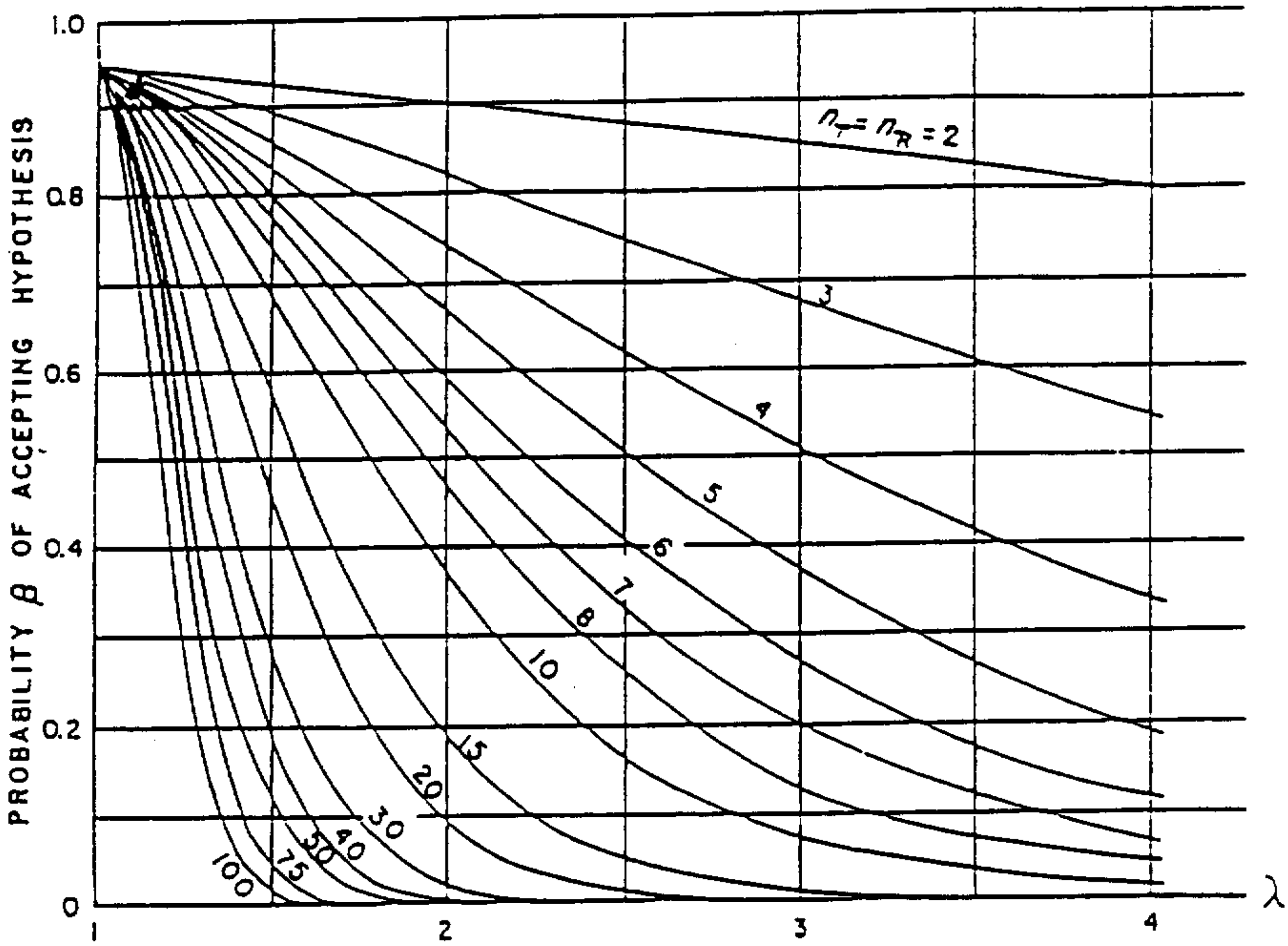


Figure 1. OC curves for testing the hypothesis
Ho: $\lambda = \sigma_T/\sigma_R = 1$ against
H1: $\lambda = \sigma_T/\sigma_R > 1$ by the F-test.
Level of significance: $\alpha = 0{,}05$

(From: Crow, E.L., Davis, F.A. and Maxfield, M.W., 1960. Statistics Manual. Dover Publications Inc. (NAVORD Report 3369 - NOTS 948, U.S. Naval Ordnance Test Station)).

The value of $n_T = n_R$, corresponding to $\beta = 0{,}10$ and $\lambda = 2{,}5$, is seen to be 12, cf. Annex B, paragraph 5.2.

2.3 Ballistic match with respect to means

An equal number of rounds of the two configurations are considered: $n_T = n_R = n$.

(a) The R- and T- observations are paired:

$d_i = X_{Ti} - X_{Ri}$; $i = 1, 2, \ldots, n$.

The population variances of the d's is

$$\sigma_d^2 = \sigma_T^2 + \sigma_R^2,$$

where σ_T^2 and σ_R^2 are the population variances of the X_T's and X_R's, respectively.

The estimate of this variance is

$$s_d^2 = \frac{1}{n-1} \Sigma (d_i - \bar{d})^2,$$

where

$$\bar{d} = \frac{1}{n} \Sigma d_i$$

is the sample mean, i.e. the estimate of the population mean $\mu = \mu_T - \mu_R$ of the d's and μ_T and μ_R are the population means of the X_T's and the X_R's respectively.

The hypothesis to be tested, is H_0: $\mu = 0$ against H_1: $\mu \neq 0$, the significance level α normally being $\alpha = 0{,}05$.

The t-test (two-sided, equal-tailed) is applied.

$$t = \frac{\bar{d}}{s_d/\sqrt{n}}$$ is computed

The hypothesis H_0 is accepted, if

$$|t| \leq t_{\alpha/2},$$

and otherwise rejected, the number of degrees of freedom being $f = n - 1$.

(b) The probability of accepting H_0 as a function of $\lambda = \mu/\sigma_d$ and the number of pairs (n) can be read from the OC curves of figure 2.

For a specific value of λ, the power of the test is $1 - \beta$.

(c) When comparing propelling charges, it is necessary to distinguish between the variances characterizing the variation within a lot:

$\sigma^2_{\text{round-to-round, R}}$ and $\sigma^2_{\text{round-to-round, T}}$,

and the variances characterizing the variation from lot to lot:

$\sigma^2_{\text{lot-to-lot, R}}$ and $\sigma^2_{\text{lot-to-lot, T}}$.

If the firings were made with charges from several R-lots and several T-lots, then the techniques described above, could be used immediately.

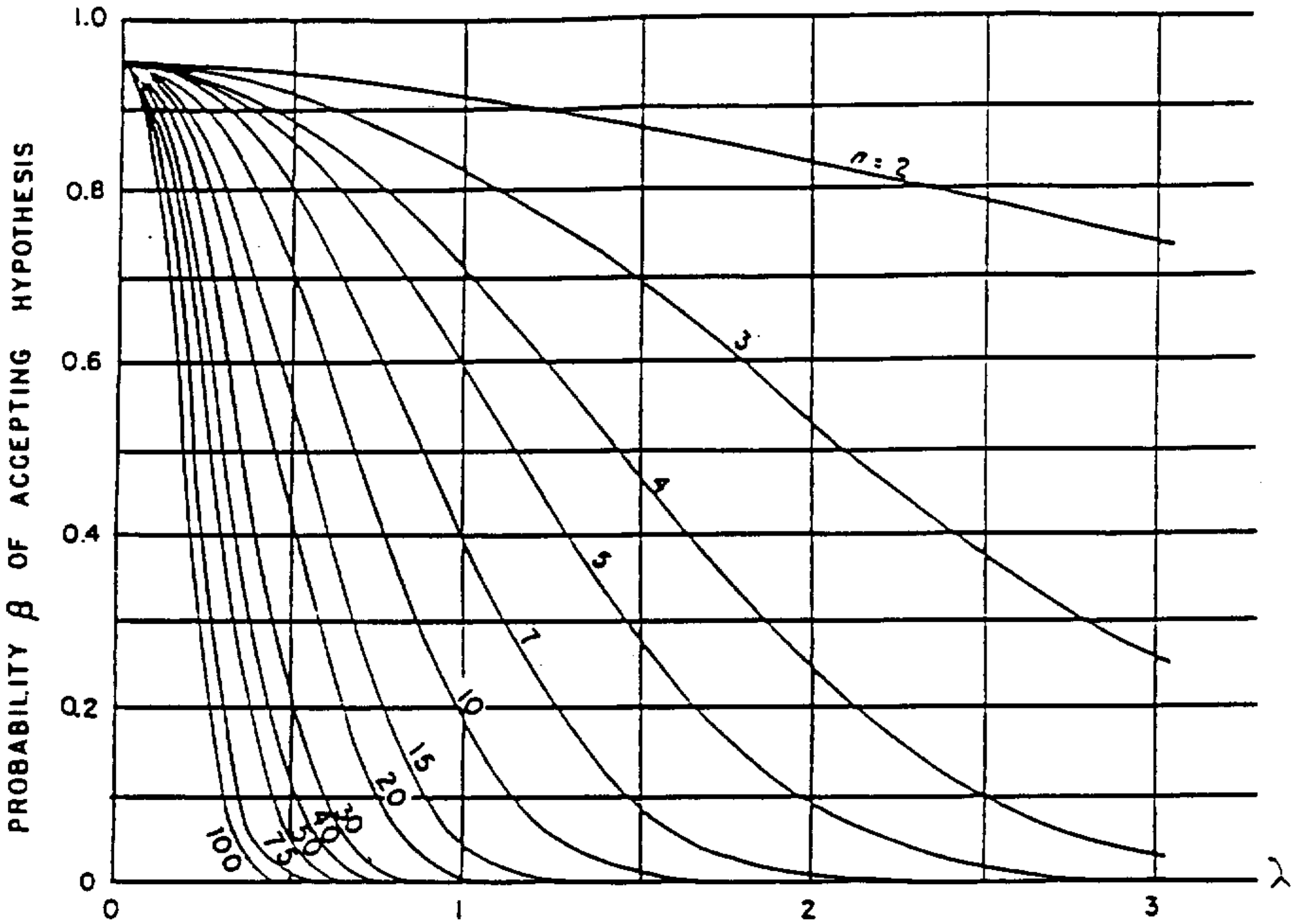


Figure 2. OC curves for testing the hypothesis Ho: $\mu = 0$, against H1: $\mu = \lambda \cdot \sigma \neq 0$ by the equal-tails t-test. Level of significance: $\alpha = 0,05$.

(From: Crow, E.L., Davis, F.A. and Maxfield, M.W., 1960. Statistics manual. Dover Publications, In.c (NAVORD Report 3369 - NOTS 948, U.S. Naval Ordnance Test Station)).

The value of n, corresponding to β = 0,10 and = 1,0, is seen to be 12, cf. Annex B, paragraph 5.2.

(d) If, however, only one R-lot and one T-lot were used, it is necessary to assume the values of lot-to-lot variances, based on estimates from other sources. The population variance of the difference between the two lot population means is

$$\sigma^2 d(\text{lot-to-lot}) = \sigma^2(\text{lot-to-lot, R}) + \sigma^2(\text{lot-to-lot, T}).$$

If the population gross mean of all R-lots is equal to the population mean of all T-lots, and if the lots to be tested are chosen at random, then the population difference

$$\mu = \mu T - \mu R$$

between the population means of the selected lots, will be normally distributed, with zero mean and the above variance.

The probability is 95%, that the two lots (R and T), chosen for test, have been selected such, that the absolute value of the population difference μ is smaller than

$$\mu_{0,975} \cdot \sigma d(\text{lot-to-lot}) = 1{,}96 \cdot \sigma d(\text{lot-to-lot})$$

("μ" indicating the cumulative normal distribution funtion).

Consequently, in this case, the acceptance interval for H_o should be extended by this quantity.

3. BALLISTIC SIMILITUDE

3.1 Introduction

An equal number of rounds (n) are fired with each configuration.

Regression analysis is used to assess similitude, based on range and deflection differences of paired observation.

(a) Range

Observed range differences (ΔR), corrected to a common MV and a common shell mass, are converted to differences (expressed in percent) in:

- form factor (Δi); or
- drag factor (Δf_D); or
- percentage of air density ($\Delta \rho$);

using:

$$\Delta i = \frac{\Delta R}{(\Delta R/\Delta i)}$$

$$\Delta f_D = \frac{\Delta R}{(\Delta R/\Delta f_D)}$$

$$\Delta \rho = \frac{\Delta R}{(\Delta R/\Delta \rho)}$$

(b) Deflection

Observed deflection differences ($\Delta defl$ (in mils)) are converted to differences (expressed in percent) in:

- lift factor (Δf_L); or
- percentage of drift ($\Delta drift$),

using:

$$\Delta f_L = \frac{\Delta defl}{(\Delta defl/\Delta f_L)}$$

$$\Delta drift = \frac{\Delta defl}{0{,}01 \, . \, drift}$$

(c) Model

The analysis is based on a linear regression model

$y = \beta_o + \beta_1 \, . \, x,$

where:

x corresponds to elevation;

y corresponds to Δi, Δf_D, $\Delta \rho$, Δf_L or drift; and

the set of coefficients (β_o, β_1) corresponds to a particular charge.

3.2 Estimation of regression coefficients

At each charge to be considered, one or more R-T pairs are fired at each of three or more elevations.

The elevations are indicated by x_i ($i = 1, 2, \ldots, m$).

The values of y, obtained at elevation x_i, are indicated by y_{ij} ($j = 1, 2, \ldots, n_i$)

Total number of pairs is $n = \Sigma n_i$

In the usual procedure:

$m = 3;\ n_1 = n_2 = n_3 = 5;\ n = 15.$

In the smallest acceptable design:

$m = 5;\ n_1 = n_2 = n_3 = n_4 = n_5 = 1;\ n = 5.$

The regression coefficients are estimated as follows:

$E(\beta_1) = b_1 = \Sigma\, \Sigma\, (x_i - \overline{x})\ (y_{ij} - \overline{y}) / \Sigma\, n_i\, (x_i - \overline{x})^2$

$E(\beta_o) = b_o = \overline{y} - b_1\overline{x},$

where:

$$\bar{x} = \frac{1}{n} \Sigma\, ni\, xi$$

$$\bar{y} = \frac{1}{n} \Sigma \Sigma\, yij$$

3.3 Testing the assumption of linearity

If more than one observation is obtained at one or more elevations, the goodness of fit can be examined, using the F-distribution:

The pooled estimate of the variance of the yij's is:

$$s_1^2 = \frac{1}{n - m} \Sigma \Sigma (yij - \bar{y}i)^2,$$

where:

$$\bar{y}i = \frac{1}{ni} \Sigma\, yij$$

is the sample mean for $x = xi$.

The scatter, in the vertical (y) direction, about the regression line is measured by

$$s_2^2 = \frac{1}{m - 2} \Sigma\, ni\, (\bar{y}i - (b_0 + b_1\, xi))^2.$$

The assumption of linearity is rejected on the α significance level, if

$$(s_2/s_1)^2 > F\alpha$$

the number of degrees of freedom being

$$(f_1, f_2) = ((m - 2), (n - m)).$$

Normally, $\alpha = 0{,}10$ is chosen.

In the normal procedure:

$(f_1, f_2) = (1, 12)$; $F_{0,10} = 3{,}18$.

In the smallest acceptable design, the test of linearity cannot be executed in this manner. In this instance, as in general, linearity should be tested subjectively by eye, based on a plot.

3.4 Construction of confidence interval

In order to construct the $(1 - \alpha)$ confidence interval about the whole regression line, the $(1 - \alpha)$ regression interval is determined, corresponding to selected values of x:

$$(b_0 + b_1 x) \pm \sqrt{2F\alpha}\ .\ se\ .\sqrt{\frac{1}{n} + (x - \bar{x})^2 / \Sigma\, ni\, (xi - \bar{x})^2}.$$

where the "Standard error of estimation" s_e is the square root of

$$s_e^2 = \frac{1}{n-2} \sum \sum (y_{ij} - (b_o + b_1 x_i))^2$$

$$= \frac{1}{n-2} \left[\sum \sum (y_{ij} - \bar{y})^2 - b_1^2 \sum n_i (x_i - \bar{x})^2 \right]$$

which is a measure of the scatter, in the vertical (y) direction, of the y_{ij}'s about the regression line.

The number of degrees of freedom for F are:

$(f_1, f_2) = (2, n - 2)$.

Normally, $\alpha = 0,10$ is chosen.

In the normal procedure: $(f_1, f_2) = (2,13)$; $F\alpha = 2,76$

In the smallest acceptable design: $(f_1, f_2) = 2,3$; $F\alpha = 5,46$

After the confidence intervals have been calculated for a number of x's, smooth curves are drawn through the plotted points, see figure 3.

3.5 Test of null hypothesis for coefficients (optional)

First test H_o: $\beta_1 = 0$ against H_1: $\beta_1 \neq 0$.

Consider:

$t = \frac{b_1}{s_b}$, where s_b is the square root of

$$s_b^2 = s_e^2 / \sum n_i (x_i - \bar{x})^2$$

H_o is rejected, if $t < - t\alpha/2$ or $t > t\alpha/2$.

If not rejected, the hypothesis H_o: $\beta_o \neq 0$ is tested against H_1: $\beta_o = 0$

Consider:

$$t = \frac{b_o}{s_e} \bigg/ \sqrt{\frac{1}{n} + \bar{x}^2 / \sum n_i (x_i - \bar{x})^2} .$$

H_o is rejected, if $t < - t\alpha/2$ or $t > t\alpha/2$.

The number of degrees of freedom is $f = n - 2$. Normally, $\alpha = 0,10$ is chosen.

In the normal procedure: $f = 13$; $t_{0,05} = 1,771$.

In the smallest acceptable design: $f = 3$; $t_{0,05} = 2,353$

If H_o: $\beta_o = 0$ is not rejected, then no correction is needed. However, paragraph 3.6 should always be completed.

3.6 Check of degree of similitude

A fixed value of $y = y_o$, equal to or close to y, is chosen as the simple ballistic correction (in range or deflection, respectively) for the charge in question. The degree of similitude, when this correction is applied, is assessed as follows:

3.6.1 Range

For selected values of x, values of y, corresponding to a change of ± 0.95% of the range are calculated, using:

$$\Delta y = \frac{0,0095 \; . \; Range}{(\Delta Range/\Delta y)}$$

where Δy is:

- form factor (Δi); or
- drag factor (Δf_D); or
- percentage of air density ($\Delta \rho$).

The values of $y = y_o \pm \Delta y$ are plotted, and smooth curves are drawn through the plots.

If the confidence bounds are not within these ± 1% curves, repeat for ± 4.75% using:

$$\Delta y = \frac{0,0475 \; . \; Range}{(\Delta Range/\Delta y)}$$

3.6.2 Deflection

For selected values of x, values of y, corresponding to ± 0,3% of the range are calculated, using:

$$\Delta y = \Delta f_L = \frac{3}{(\Delta drift/\Delta f_L)} \quad or$$

$$\Delta y = \Delta drift \text{ (in \% of drift)} = \frac{300}{drift}$$

The values of $y = y_o \pm \Delta y$ are plotted, and smooth curves are drawn through the plots.

If the confidence bounds are not within these ± 0,3% lines, repeat for ± 1,5% using:

$$\Delta y = \Delta f_L = \frac{15}{(\Delta drift/\Delta f_L)} \quad or$$

$$\Delta y = \Delta drift \text{ (in \% of drift)} = \frac{1500}{drift}$$

3.6.3 Degree of similitude

Similitude 1% is said to exist, if, when the selected set of simple ballistic corrections (y_o for range and deflection) is applied, the confidence interval for range is within the ± 0.95%

of range limits, and the confidence interval for deflection is within the ± 0,3% of range limits.

Similitude 5% is said to exist, if, when the selected set of simple ballistic corrections (y_0 for range and deflection) is applied, the confidence interval for range is within the ± 4.75% of range limits, and the confidence interval for deflection is within the ± 1,5% of range limits.

The general principle is illustrated in figure 3.

Generally, the curves should start with QE, corresponding to about 20% of the maximum range.

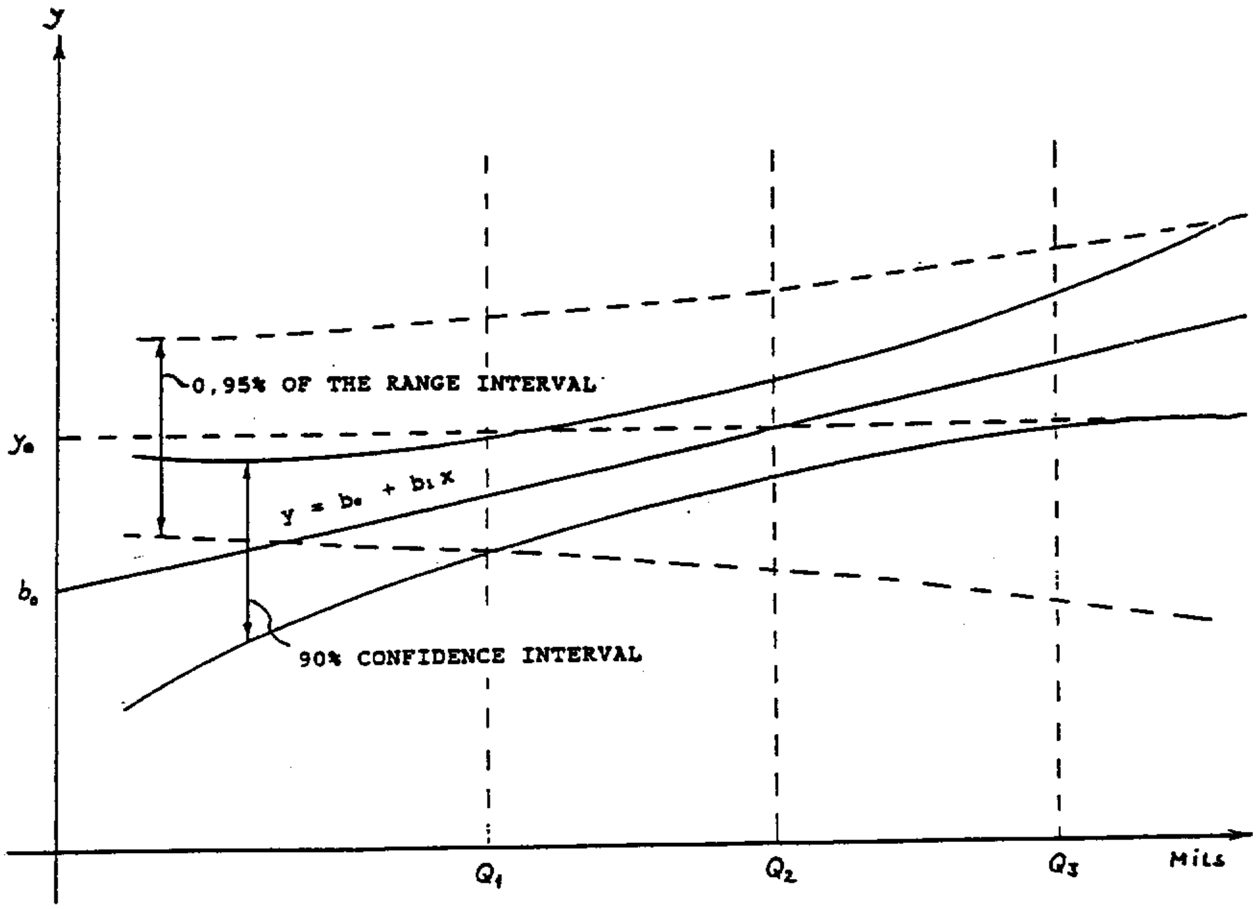


Figure 3. Regression line, 90% confidence interval, 0.95% of the range interval (principle).

In this case, it is seen that 1% similitude has been established at quadrant elevations above QE1.

4. Outliers

The following outlier test is recommended:

Grubb's outlier test(*)

Given a sample of n data: $x_1, x_2, \ldots, x_n$.

$$T = \frac{|x_k - \bar{x}|}{s}$$ is computed, where

x_k represents the largest (smallest) observation in the sample to be tested if it is an outlier,

$$\bar{x} = \frac{1}{n} \Sigma x_i$$ is the sample mean, and

$$s^2 = \frac{1}{n-1} \Sigma (x_i - \bar{x})^2$$ is the sample variance.

If T is greater than the critical value found in the table below for a certain significance level α, then x_k may be considered as an outlier.

(*) Grubbs, F.E., 1969. Procedures for Detecting Outlying Observations in Samples. Technometrics, Vol. 11, No. 1, pp. 1-21.

Table of Critical Values for T (One-sided Test) When Standard Deviation is Calculated from the Same Sample

Number of Observations n	5% Significance Level	2.5% Significance Level	1% Significance Level
3	1.15	1.15	1.15
4	1.46	1.48	1.49
5	1.67	1.71	1.75
6	1.82	1.89	1.94
7	1.94	2.02	2.10
8	2.03	2.13	2.22
9	2.11	2.21	2.32
10	2.18	2.29	2.41
11	2.23	2.36	2.48
12	2.29	2.41	2.55
13	2.33	2.46	2.61
14	2.37	2.51	2.66
15	2.41	2.55	2.71
16	2.44	2.59	2.75
17	2.47	2.62	2.79
18	2.50	2.65	2.82
19	2.53	2.68	2.85
20	2.56	2.71	2.88
21	2.58	2.73	2.91
22	2.60	2.76	2.94
23	2.62	2.78	2.96
24	2.64	2.80	2.99
25	2.66	2.82	3.01
30	2.75	2.91	
35	2.82	2.98	
40	2.87	3.04	
45	2.92	3.09	
50	2.96	3.13	
60	3.03	3.20	
70	3.09	3.26	
80	3.14	3.31	
90	3.18	3.35	
100	3.21	3.38	

(*) Grubbs, F.E., 1969. Procedures for Detecting Outlying Observations in Samples. Technometrics, Vol. 11, No. 1, pp. 1-21.

DETERMINATION OF THE REQUIRED SAMPLE SIZE FOR FALL OF SHOT METHOD

1. Guidelines for the determination of the required sample size can be based on statistical considerations, on practical considerations (cost, economy, time limitation and so on) or on both.

2. Considerations of pure statistical nature

For this case two types of tests of hypothesis are foreseen

- test on equality of VARIANCES
- test on equality of MEANS (most important test)

The required sample size will be based on the test of hypothesis of MEANS, namely:

H_0 :	$\mu_T = \mu_R$	equality of two means (null hypotheses)
H_1 :	$\mu_T \neq \mu_R$	true means are significantly different (alternative hypothesis)

This is a 2-sided test.

Let α be the significance level of the test, and $\sigma_T = \sigma_R$ but unknown (tested by a test of hypothesis on variances).

Two practical approaches are possible (based on statistical considerations).

2.1 1st approach

One determines the sample sizes n_T (for the test configuration) and n_R (for the reference configuration) in such a way that the OC curve (operating characteristic curve) of the test goes through the two points A and B corresponding to the "interests" of the two parties involved ("producer" and "consumer").

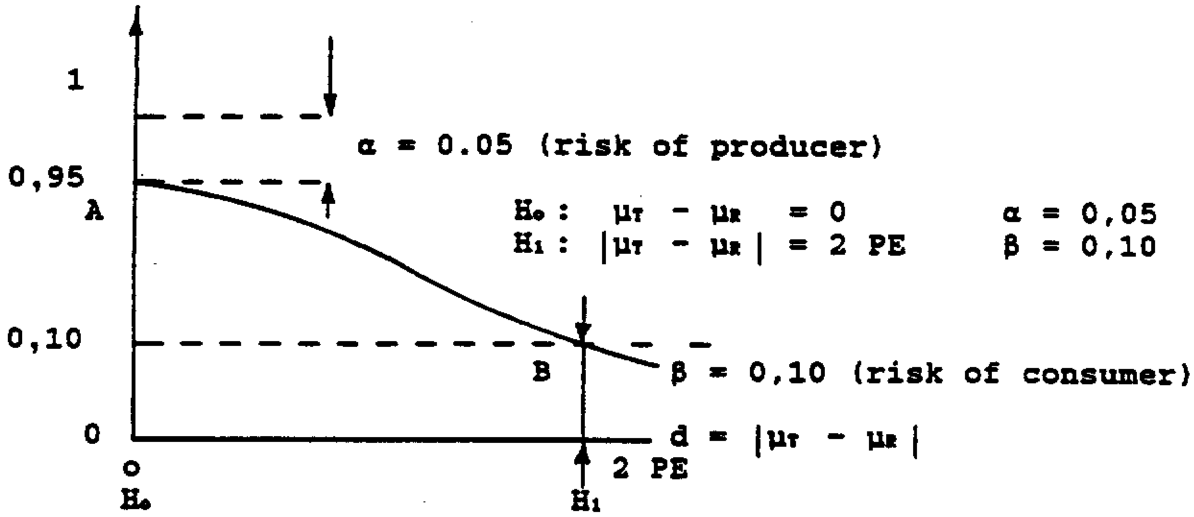
H_0 (null-hypotheses): $\mu_T - \mu_R = 0$ with significance level α

H_1 (alternative hypothesis): $|\mu_T - \mu_R| =$ specific value with β = probability to fail to detect that specific value

A common value for α is 0,05 and for β is 0,10.

Suppose H_1 : $|\mu_T - \mu_R| = 2$ PE

$\beta = p$ (accept H_0) $= p$ (fail to detect $d = |\mu_T - \mu_R|$)



In fact, it means that the design of the sample size meets the risk the two parties, "producer" and "consumer", are willing to take.

α = 0,05 is the risk of committing a type-1-error, meaning that the producer has a risk of seeing rejected the null hypothesis H_0 although it was right ($\mu_T = \mu_R$).

Put in another way, the "producer" expects that with the chosen sample sizes, the test of hypothesis will result in 95% of all cases, in the acceptance of H_0, but, unfortunately, in 5% of the cases, it will erroneously lead to rejection of the correct H_0.

β = 0,10 is the risk of committing a type-2-error, meaning that the "consumer" - here the potential buyer, user or the artillery man - has a risk of 10% of accepting the H_0 ($\mu_T = \mu_R$) whereas it should have been rejected because an alternative hypothesis H_1 ($|\mu_T = \mu_R|) \geq 2$ PE) is true. This 2 PE is what the "consumer" is willing to accept at the limit with 10% chance.

So, most of the time, namely in 90% of all cases, the sampling plan will lead to reject H_0 correctly, because it was an incorrect hypothesis, but in 10% of the cases, he erroneously accepts the H_0.

In order to find n_T and n_R, tables or OC-curves can be consulted.

For this typical case: $H_0 : |\mu_T - \mu_R| = 0$ $\alpha = 0,05$

$H_1 : |\mu_T - \mu_R| = 2$ PE $\beta = 0,10$

one obtains $n_T = n_R = n = 12$

Remarks:

- The value $|\mu_T - \mu_R| = 2$ PE was based upon terminal ballistic considerations, namely that a systematic offset (bias) of 2 PE starts yielding a significant degradation (typically around 10% for a lot of artillery weapon-systems at typical ranges) in terminal ballistics performance.
- The best power of the test $(1 - \beta)$ is obtained when $\mu_T = \mu_R$.

Of course, one is "free" to chose other sets of risks α and β, but α, β and n (sample size) are not independant of each other. In fact, if n (the common sample size is maintained at a fixed value), decreasing one type risk automatically increases the other type risk.

For example, if for the α chosen (suppose α = 0,05), the sample size required to achieve a reasonably small β (suppose β = 0,10), is too large (suppose n = 12), and it is really important to detect a minimum difference of 2 PE - when it exists - then a less conservative (i.e. larger) value of α (suppose α = 0,10) could be proposed, which will reduce the sample size to n = 8 in order to get the same risk β = 0,10 for the consumer. But, then the "producer" must be willing to take a higher risk (of 10% instead of 5%).

2.2 2nd approach

One can fix a value for the common sample size $n_T = n_R = n$ and then verify what are the potential risks for the two parties involved. In the following tables, the values for 1 - β or β displayed for a certain number of values for n and the three values for α, namely 0,01, 0,05 and 0,10.

$1 - \beta$ = p (detect a difference $|\mu_T - \mu_R|$ = 2 PE
= p (reject an incorrect H_0)

n	α = 0,01	0,05	0,10
3			0,50
4			0,64
5		0,50	0,73
6		0,60	0,80
7		0,65	0,85
8	0,50	0,70	0,90
9	0,55	0,80	0,92
10	0,60	0,83	0,94
11	0,70	0,86	0,95
12	0,75	0,90	
13	0,80	0,93	0,99
14	0,83	0,95	
15	0,86		
16	0,90		
17	0,92		
18	0,93		
19	0,95		
20		0,99	
..			
26	0,99		

β = p (detect a difference $|\mu_T - \mu_R|$ = 2 PE
= p (reject an incorrect H_o)

n	α = 0,01	0,05	0,10
3			0,50
4			0,37
5		0,50	0,27
6		0,40	0,20
7		0,35	0,15
8	0,50	0,30	0,11
9	0,45	0,20	0,08
10	0,40	0,17	0,06
11	0,30	0,14	0,05
12	0,25	0,10	
13	0,20	0,07	0,01
14	0,17	0,05	
15	0,14		
16	0,10		
17	0,08		
18	0,07		
19	0,05		
20		0,01	
..			
26	0,01		

When the required sample sizes are determined in order to perform a convenient test of hypothesis on the equality of the true means, then one can have an idea how good will be the one-sided test on equality of variances (that should precede the test on the means) with the same sample sizes as determined before.

In this case: H_o : $\sigma_T^2 = \sigma_R^2$ or $\frac{\sigma_T^2}{\sigma_R^2} = 1$ with α

H_1 : $\mu_T^2 = k^2 \cdot \mu_R^2$ or $\frac{\mu_T^2}{\mu_R^2} = k^2 > 1$ with β

The table below gives the values for β = p (fail to detect a $\sigma_T = k . \sigma_R$)

n	α = 1,5	α = 2	α = 2,5	α = 3	α = 3,5
3	0,92	0,83	0,75	0,68	0,62
4				0,50	
5			0,50		
6					
7		0,50			0,10
8					
9				0,10	
10					
11					
12			0,10		
..					
18	0,50				
19					
20		0,10			

3. Considerations of practical nature

Due to practical constraints (costs, economy, duration of the test, ...) one has not always the possibility to determine the required sample sizes based upon the guidelines dictated by pure statistical considerations. Referring to the previous table for testing the means, one finds that (for α = 0,05) $n_T = n_R = n = 5$ corresponds to $\beta = 1 - \beta = 0,5$ (break-even-point): in 50% of the cases, the consumer thinks the true means are equal ($\mu_T = \mu_R$), but in reality $| \mu_T - \mu_R | >$ 2 PE, what he was not willing to tolerate.

So one could argue that $n_T = n_R = n = 5$ is about to be the minimum sample size, which however constitutes a large (50%) risk for the "consumer" or artillery man.

In fact, when other reasons than statistical ones have yielded the sample size, the decision-maker should always keep in mind that too small sample sizes yield very large risks for the "consumer". If however one has to make a compromise, one should try to propose sample sizes that are closer to about n = 12 (good guarantee of the consumer's risk) that to n = 5 which gives a really poor guarantee for the "consumer", provided α = 0,05.

PROCEDURE BASED ON CALIBRATED MODIFIED POINT MASS TRAJECTORY AND RADAR MEASUREMENTS

1. AIM

1.1 The substance of this annex is to determine by using Radar Techniques or Simulation Techniques, whether the external ballistic performance of a shell in a test configuration (T) can be considered as matched or similar to the external ballistic performance of a shell in a reference configuration (R). The simple external ballistic corrections, as described in Annex F, can be determined, if required.

2. CORRECTIONS

2.1 In the case of ballistic similitude, the simple external ballistic corrections considered are simple adjustments to the modified point mass (MPM) trajectory model data (STANAG 4355). The corrections considered are for the reference configuration in the form of drag, lift or magnus force multiplying factors which may be constants or functions of the quadrant elevation (in the case of fire control computers). Such corrections can be applied to the firing table, through density, drift or time of flight corrections.

3. PROCEDURES

3.1 General Remarks

3.1.1 To be considered as a reference configuration (R), a substantial package of information must be available for that configuration. This will consist of:

(a) the trajectory model (STANAG 4355), with its aerodynamic data and its associated calibration factors for each gun from which it has been fired;

(b) firing table (STANAG 4119);

(c) full range and accuracy firing reports with associated analysis.

3.1.2 A similar package of data should be available for the test configuration (T) but at least an external ballistic assessment should have been carried out. A simulation procedure or a firing procedure can be selected depending on the quality of the data available for the (T) configuration or the importance of the particular R/T combination from the user viewpoint (see Figure 1).

3.1.3 The steps applying the procedures in this annex are outlined in Figure 1. The initial step in the process is to examine the existing data and to determine whether match or similitude can be determined from simulation alone or whether trials are required. This study should be carried out by ballisticians and user representatives with experience of large calibre weapons. The simulation procedure is described in para. 3.2 and the firing procedure in para. 3.3.

3.2 Simulation Procedure

This procedure can be used to determine the degree of similarity between well known (R) and (T) configurations. In this case, the data package for the (R) and (T) configurations will consist, at least of:

(a) the MPM aerodynamic and inertia data together with the associated calibration factors (STANAG 4355);

(b) firing table (STANAG 4119);

(c) full Range and Accuracy firing reports with associated analysis;

(d) the (T) configuration has been fired to demonstrate feasibility.

It can also be used where the user/ballistician teams determine that a particular combination of R/T has a low priority (very unlikely to occur) and that sufficient information is available to assign the combination to the appropriate degree of interchangeability without verification by trials.

Note that the calibrated model represents the best estimate of the performance of a configuration for all muzzle velocities and elevations. Individual solutions may contain bias errors.

3.2.2 The degree of similarity between the external ballistic performances of the two configurations (R) and (T) will be determined by simulation using the following steps:

(a) simulate the (R) configuration with the R data MPM model for a selection of representative initial conditions of charges andelevations, yielding an estimate of the population means in range and drift;

simulate the (T) configuration with the T data MPM model for a selection of representative initial conditions of charges and elevations, yielding an estimate of the population means in range and drift;

(b) compare the results of the simulations;

(c) if none of the differences in range and drift between (R) and (T) simulations are greater than one probable error in range and drift, as taken from the (R) Firing Table, then ballistic match is achieved;

(d) if none of the differences in range and drift between (R) and (T) simulations, either with or without corrections, are greater than 0.95%/4.75% of range in range and and 0,3%/1,5% of range in drift, then ballistic similitude (1%/5%) can be achieved. If correction factors are required, they shall be reported in the format of Annex F;

(e) if after applying corrections there are differences in range or drift between (R) and (T) simulations greater than 4.75%/1,5% of range in range and drift, then ballistic similitude cannot be achieved;

(f) if necessary, confirmatory firings can be performed.

3.3 Firing Procedure

3.3.1 This procedure can be used to determine ballistic match or the degree of similarity between (R) and (T) configurations. Examples are:

(a) the reference shell with its reference fuze (R) and the same shell with another fuze (T);

(b) the reference shell designed and produced in one country (R) and the same shell produced in another country (T);

(c) the reference shell with a reference payload (R) and the same shell with another payload (T).

3.3.2 The Doppler Radar systems available can deliver measurements of radial velocity directly. These can be used with wind and air density measurements together with the trajectory model (STANAG 4355), to yield drag force and hence drag coefficient (C_D) as a function of Mach number. Note that this is total C_D and includes the effect of any yaw present (STANAG 4355).

Position radars can in addition to allowing one to estimate drag force differences, also provide the data needed to estimate lift and magnus force corrections.

3.3.3 Both configurations must be fired under identical conditions on the same occasion from the type of equipment to be used in service. The elevations chosen will be sufficiently high to give reasonable Mach number coverage throughout flight, but no higher. This reduces the requirement for higher altitude wind measurements and gives higher rates of fire and better range usage. The charges and elevations chosen should ensure a complete Mach number coverage for each configuration fired.

3.3.4 The firing trial will be conducted using (R) and (T) configurations alternately at each of the charges required. The number of rounds fired in each serial should be determined by the level of confidence required for match or similitude and may be determined from any standard statistical test method which may be selected from Annex D. Depending on the trial design selected from Figure 1, the minimum number of rounds is given in Table 1.

3.3.5 The (T) configuration results can be used to produce simple corrections which can be incorporated with the (R) model to give an adjusted model, say Adjusted Reference model (AR). If correction factors are required, they shall be reported in the format of Annex F.

3.3.6 Ballistic match or the level of similitude, or indeed failure to achieve either, will be determined by simulating firing conditions according to para. 3.2 by replacing the (R) model with the (AR) model. The level of confidence achieved should be determined by confirmatory firings as in para. 3.2.2.f.

3.3.7 The fall of shot data are recommended to be collected and analysed according to guidelines in Annex C as a routine part of radar procedure.

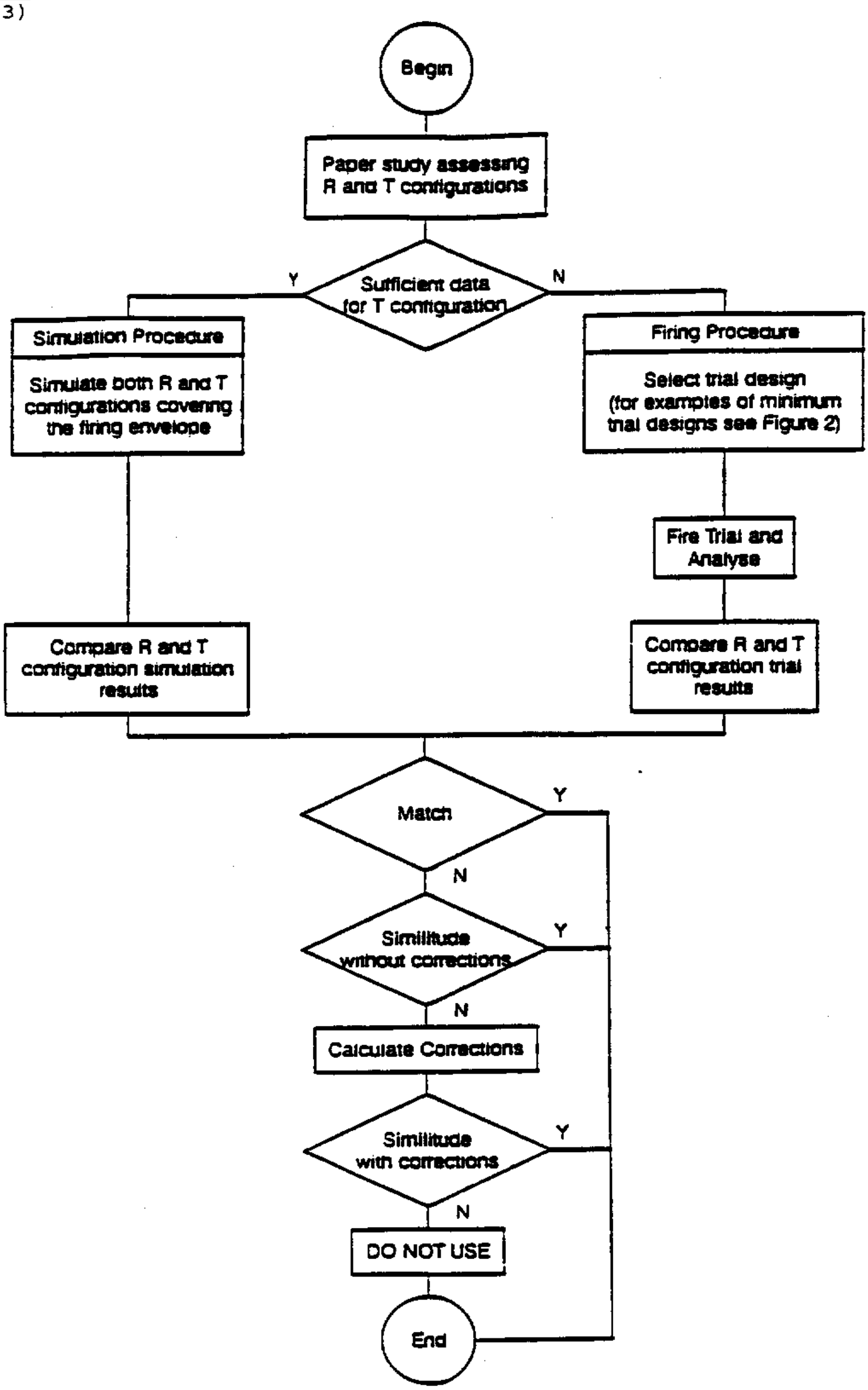
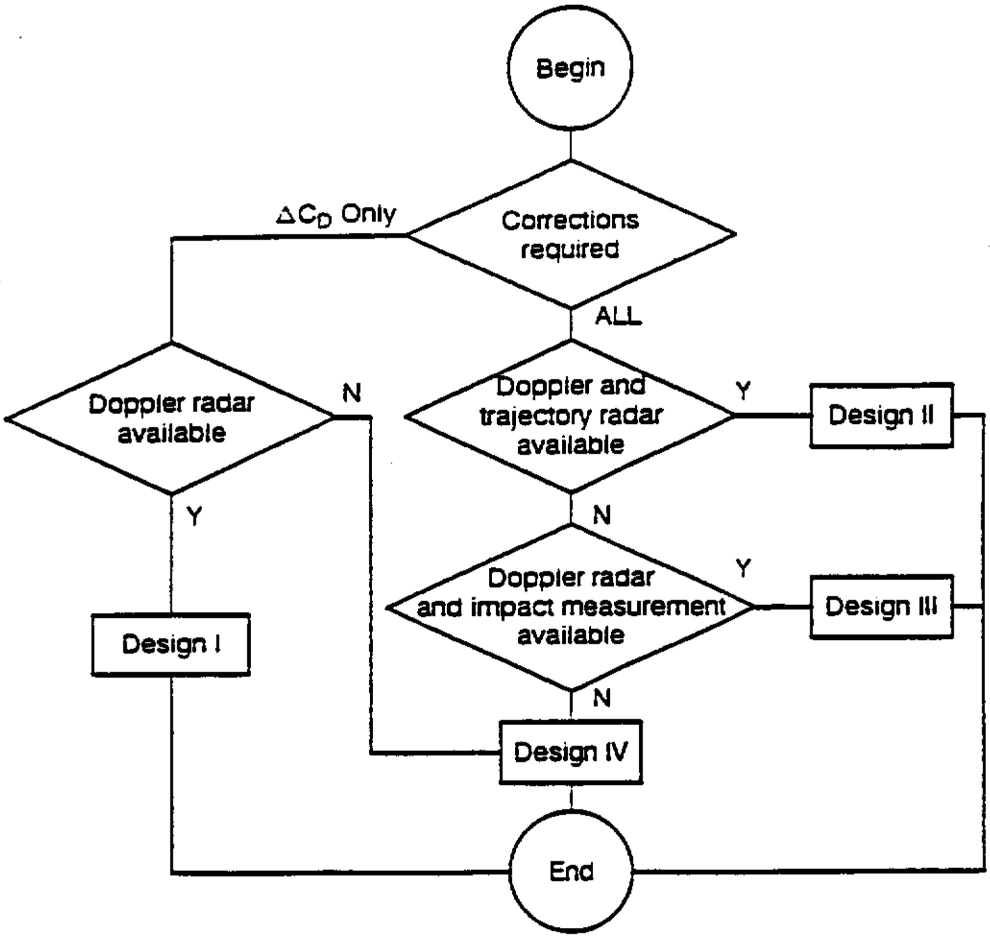


Figure 1. Procedure to determine the ballistic similarity of two configurations.



Note. The minimum trial designs are given in Table 1

Figure 2.   Examples of trial designs

Design	Purpose	Instrumentation	Elevation mils *)	Charges					Minimum no. of rounds
				L	I_1	I_2	I_3	H	
I	To determine ΔC_D only where ΔC_L is expected to be zero	Doppler radar	400	1R/1T		1R/1T		1R/1T	9R/9T
			700	1R/1T		1R/1T		1R/1T	
			1100**)	1R/1T		1R/1T		1R/1T	
II	To determine all correction factors (ΔC_D, ΔC_L, Δ Magnus force, ΔMV)	Doppler- and trajectory radar	400	1R/1T	1R/1T	1R/1T	1R/1T	1R/1T	15R/15T
			700	1R/1T	1R/1T	1R/1T	1R/1T	1R/1T	
			1100**)	1R/1T	1R/1T	1R/1T	1R/1T	1R/1T	
III	To determine all correction factors (ΔC_D, ΔC_L, Δ Magnus force, ΔMV)	Doppler radar and impact measurement	350	1R/1T		1R/1T		1R/1T	15R/15T
			550	1R/1T		1R/1T		1R/1T	
			750	1R/1T		1R/1T		1R/1T	
			950**)	1R/1T		1R/1T		1R/1T	
			1150**)	1R/1T		1R/1T		1R/1T	
			With velocimeter, this design also meets the minimum requirements for the fall of shot method (Annex B)						
IV	To determine all correction factors	Impact measurement (The Fall of Shot Method, Annex B)	400	5R/5T		5R/5T		5R/5T	45R/45T
			700	5R/5T		5R/5T		5R/5T	
			1100**)	5R/5T		5R/5T		5R/5T	

*) All elevations are approximate
Elevations (and charges) may vary to meet radar requirements.
**) Only when high elevation is required.
1R/1T 1 reference and 1 test configuration.

L Lowest charge.
H Highest charge.
I_1, I_2, I_3 Intermediate charges.

Table 1. Minimum trial designs for the firing procedure

REPORT ON FINDINGS

1. Any report on the findings to be used by other NATO nations should contain the following information:

- The weapon and ammunition combinations that have been investigated.
- The comparison procedure that has been followed:
  - Fall of shot method (Annex C)

  or
  - Doppler radar method (Annex D)
- The degree of similarity of the ballistic performance, plus the applicable reference to aiming data equivalent to the estimated degree of similarity.
- The interval of quadrant elevations that has been used.
- A figure to illustrate the results (of figure 3, Annex C).

2. A format for simple ballistic corrections is given below.

FORMAT FOR SIMPLE BALLISTIC CORRECTIONS

A difference in		Muzzle velocity	Range	Drift	Time of flight
is compensated for by applying a correction to	either	Muzzle velocity (meters/second)	Air density (percent)	Correction for drift (percent)	Time of flight (second or percent)
	or	Burning Rate Factor: f_β and Down-tube Resistance Factor: f_R (percent)	Form Factor: i or Drag Factor: f_D (percent)	Lift Factor: f_L (percent)	Magnus Force Factor: Ω_M (percent)
Projectile	Charge				